

Generalized complex marginal deformation of pp-waves and giant gravitons

Sunyoung Shin*

Bogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Moscow region, Russia

Abstract

We present the Penrose limits of a complex marginal deformation of $AdS_5 \times S^5$, which incorporates the $SL(2, \mathbb{R})$ symmetry of type IIB theory, along the $(J, 0, 0)$ geodesic and along the (J, J, J) geodesic. We discuss giant gravitons on the deformed $(J, 0, 0)$ pp-wave background.

*e-mail : shin@theor.jinr.ru

1 Introduction

The marginal deformation [1] introduces phases in the superpotential which breaks the $SO(6)_R$ R-symmetry group to its $U(1) \times U(1) \times U(1)_R$ Cartan subgroup. In the gravity side [2], the $U(1) \times U(1)$ non-R-symmetry maps to a two-torus. The dual geometry is obtained by applying an $SL(2, \mathbb{R})$ transformation which acts on the Kähler modulus of the corresponding two-torus or equivalently a TsT (T-duality, shift, T-duality) transformation. The phases in the gauge theory can be complexified. In the dual geometry, it corresponds to a specific $SL(3, \mathbb{R})$ transformation which consists of the $SL(2, \mathbb{R})$ transformation and an S-duality transformation $SL(2, \mathbb{R})_s$ or equivalently an STsTS (S-duality, T-duality, shift, T-duality, S-duality) transformation [2, 3].¹ The three-parameter generalization is proposed as a dual geometry to a non-supersymmetric marginal deformation of $\mathcal{N} = 4$ super Yang-Mills theory [3].

The charges of chiral superfields under the $U(1) \times U(1)$ symmetry in the gauge theory corresponds to the angular momenta along the two-torus in the dual geometry. In terms of the angle coordinates (ϕ_1, ϕ_2, ϕ_3) of S^5 , there are four possible BPS geodesics, $(J_{\phi_1}, J_{\phi_2}, J_{\phi_3}) \sim (J, 0, 0)$, $(0, J, 0)$, $(0, 0, J)$ and (J, J, J) . The Penrose limit along the first three geodesics and the Penrose limit along the fourth geodesic are two distinct pp-waves. The pp-waves are discussed in [4, 2, 5, 14].

A point graviton which has an angular momentum about the sphere of $AdS_m \times S^n$ blows up into a spherical brane [6]. A giant graviton is a spherical $(n - 2)$ -brane which wraps a part of S^n . A dual giant graviton is a spherical $(m - 2)$ -brane which wraps a spatial part of AdS_m . Both are BPS objects, which have the same quantum numbers as the Kaluza-Klein mode of the point graviton [7, 8]. Giant gravitons in the Penrose limit of $AdS_5 \times S^5$ are studied in [9].

Giant gravitons on the three-parameter non-supersymmetric background [3] are discussed in [10, 11]. It is shown in [11] that the (dual) giant gravitons do not depend on the deformation parameters γ_i , $(i = 1, 2, 3)$. (Dual) giant gravitons in the supersymmetric deformation are obtained by setting $\gamma_i = \gamma$. D3-brane (dual) giant gravitons and D5-brane dual giant gravitons on γ -deformed $AdS_5 \times S^5$ are discussed in [12]. Giant gravitons in the Penrose limits of marginally deformed $AdS_5 \times S^5$ along the $(J, 0, 0)$ geodesic and along the (J, J, J) geodesic are considered in [13, 14]. It is shown in [13] that the giant graviton on the deformed $(J, 0, 0)$ pp-wave is independent of the deformation parameter γ and energetically degenerate with the Kaluza-Klein point graviton whereas the giant graviton on the deformed (J, J, J) pp-wave does not retain its round three-sphere shape. In [14], the Penrose limits of the complex marginal deformation of $AdS_5 \times S^5$ along the $(J, 0, 0)$ geodesic and along the (J, J, J) geodesic are studied. Giant gravitons and dual giant gravitons are discussed on the deformed $(J, 0, 0)$ pp-wave. It is shown that the giant gravitons are not energetically degenerate with the point graviton and exist only up to a critical value of σ . They are energetically unfavorable but nevertheless perturbatively stable.

¹ γ is used for the $SL(2, \mathbb{R})$ transformation and σ is used for the $SL(2, \mathbb{R})_s$ transformation. Both are real parameters with unit period.

In this work, we study the Penrose limits of complex marginal deformation of $AdS_5 \times S^5$, which incorporates the $SL(2, \mathbb{R})$ symmetry of type IIB theory and observe giant gravitons on the deformed $(J, 0, 0)$ pp-wave background. In section 2, we review the generalized complex marginal deformation of $AdS_5 \times S^5$ [15, 16], and present the pp-wave geometries which are obtained by taking the Penrose limits along the $(J, 0, 0)$ geodesic and along the (J, J, J) geodesic. In section 3, we study the giant graviton solution on the pp-wave background and check the stability by observing small fluctuations about the solution. In section 4, we summarize our results.

2 Generalized complex marginal deformation

The Lunin-Maldacena $SL(3, \mathbb{R})$ transformation, which generates the gravity dual of the complex marginal deformation [1, 2] is

$$\Lambda_{LM}^T = \begin{pmatrix} 1 & 0 & 0 \\ \gamma & 1 & \sigma \\ 0 & 0 & 1 \end{pmatrix}. \quad (2.1)$$

The transformation can be generalized by an $SL(3, \mathbb{R})$ transformation

$$L = \begin{pmatrix} L_{11} & 0 & L_{13} \\ 0 & 1 & 0 \\ L_{31} & 0 & L_{33} \end{pmatrix}, \quad \det L = 1, \quad (2.2)$$

which corresponds to the $SL(2, \mathbb{R})$ symmetry of type IIB supergravity. The $SL(3, \mathbb{R})$ transformation $L\Lambda_{LM}^T$, therefore produces a generalized complex marginal deformation [2, 15]. We consider $AdS_5 \times S^5$ defined by

$$ds^2 = R^2 \left[-dt^2 \cosh^2 \rho + d\rho^2 + \sinh^2 \rho d\Omega_3^2 + \sum_{i=1}^3 d\mu_i^2 + \sum_{i=1}^3 \mu_i^2 d\phi_i^2 \right],$$

$$\begin{aligned} \chi_0 &= \tau_1, \quad e^{-\Phi_0} = \tau_2, \quad B_2 = 0, \quad C_2 = 0, \\ C_4 &= 4R^4 e^{-\Phi_0} (\omega_4 + \omega_1 \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3), \\ F_5 &= 4R^4 e^{\Phi_0} (\omega_{AdS_5} + \omega_{S^5}), \\ \omega_{AdS_5} &= d\omega_4, \quad \omega_{S^5} = d\omega_1 \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3, \\ d\omega_1 &= \cos \alpha \sin^3 \alpha \cos \theta \sin \theta d\alpha \wedge d\theta, \\ \mu_1 &= \cos \alpha, \quad \mu_2 = \sin \alpha \cos \theta, \quad \mu_3 = \sin \alpha \sin \theta, \end{aligned} \quad (2.3)$$

where R is the radius of AdS_5 and the radius of S^5 . The complex marginal deformation of $AdS_5 \times S^5$ [16] is

$$\begin{aligned}
ds^2 &= R^2 H^{1/2} \left[-dt^2 \cosh^2 \rho + d\rho^2 + \sinh^2 \rho d\Omega_3^2 + \sum_{i=1}^3 (d\mu_i^2 + G\mu_i^2 d\phi_i^2) \right. \\
&\quad \left. + G\mathcal{P}\mu_1^2\mu_2^2\mu_3^2 \left(\sum_{i=1}^3 d\phi_i \right)^2 \right], \\
e^\Phi &= \sqrt{G}H\tau_2^{-1}, \\
\chi &= H^{-1} (h + \tau_2^2 \hat{\gamma} \hat{\sigma} g_0), \\
B_2 &= R^2 G\mathcal{Q}\omega_2 - 4R^2 \tau_2 \hat{\sigma} \omega_1 \wedge \sum_{i=1}^3 d\phi_i, \\
C_2 &= R^2 G\mathcal{T}\omega_2 - 4R^2 \tau_2 \hat{\gamma} \omega_1 \wedge \sum_{i=1}^3 d\phi_i, \\
C_4 &= 4R^4 \tau_2 \omega_4 + 4R^4 \tau_2 G \left[1 - \hat{\sigma} \mathcal{T} g_0 \right] \omega_1 \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3, \\
F_5 &= 4R^4 \tau_2 (\omega_{AdS_5} + G\omega_{S^5}), \tag{2.4}
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{P} &= \hat{\gamma}^2 f - 2\hat{\gamma} \hat{\sigma} h + \hat{\sigma}^2 g, \\
\mathcal{Q} &= \hat{\gamma} f - \hat{\sigma} h, \\
\mathcal{T} &= \hat{\gamma} h - \hat{\sigma} g, \tag{2.5}
\end{aligned}$$

$$\begin{aligned}
G^{-1} &= 1 + \mathcal{P}g_0, \\
H &= f + \tau_2^2 \hat{\sigma}^2 g_0, \\
g_0 &= \mu_1^2 \mu_2^2 + \mu_2^2 \mu_3^2 + \mu_3^2 \mu_1^2, \\
\omega_2 &= \mu_1^2 \mu_2^2 d\phi_1 \wedge d\phi_2 + \mu_2^2 \mu_3^2 d\phi_2 \wedge d\phi_3 + \mu_3^2 \mu_1^2 d\phi_3 \wedge d\phi_1, \tag{2.6}
\end{aligned}$$

and

$$\begin{aligned}
f &= (L_{33} + L_{13}\tau_1)^2 + L_{13}^2 \tau_2^2, \\
g &= (L_{31} + L_{11}\tau_1)^2 + L_{11}^2 \tau_2^2, \\
h &= (L_{33} + L_{13}\tau_1)(L_{31} + L_{11}\tau_1) + L_{11}L_{13}\tau_2^2. \tag{2.7}
\end{aligned}$$

The $SL(2, \mathbb{R})$ transformation (2.2) can be identified with torus parameters from an eleven dimensional viewpoint. The parametrization considered in [15, 16] is

$$L_{11} = 1, \quad L_{13} = \frac{r_3}{R_1} \cos \xi, \quad L_{31} = 0, \quad L_{33} = 1, \tag{2.8}$$

with a constraint

$$r_3 = \frac{R_3}{\sin \xi}. \quad (2.9)$$

R_i , ($i = 1, 3$) are the torus radii before the torus deformation and r_3 is the torus radius of the third direction after the deformation. ξ is the intersection angle between the direction along the first direction and the direction along the third direction. The geometry can be simplified by identifying the axion-dilaton coupling with the torus modulus of the rectangular torus before the torus deformation as

$$\tau = \tau_1 + i\tau_2 = il, \quad l := \frac{R_1}{R_3}. \quad (2.10)$$

The deformed $AdS_5 \times S^5$ is [15]

$$\begin{aligned} ds^2 &= R^2 \tilde{H}^{1/2} \left[-dt^2 \cosh^2 \rho + d\rho^2 + \sinh^2 \rho d\Omega_3^2 + \sum_{i=1}^3 \left(d\mu_i^2 + \tilde{G} \mu_i^2 d\phi_i^2 \right) + 9\tilde{G}\tilde{\mathcal{P}} \mu_1^2 \mu_2^2 \mu_3^2 d\psi^2 \right], \\ e^\Phi &= \sqrt{\tilde{G}\tilde{H}} l^{-1}, \\ \chi &= \tilde{H}^{-1} (l \cot \xi + \hat{\gamma} \hat{\sigma} l^2 g_0), \\ B_2 &= R^2 \tilde{G} \tilde{\mathcal{Q}} \omega_2 - 4R^2 \hat{\sigma} l \omega_1 \wedge \sum_{i=1}^3 d\phi_i, \\ C_2 &= R^2 \tilde{G} \tilde{\mathcal{T}} \omega_2 - 4R^2 \hat{\gamma} l \omega_1 \wedge \sum_{i=1}^3 d\phi_i, \\ C_4 &= 4R^4 l \omega_4 + 4R^4 l \tilde{G} (1 - \hat{\sigma} \mathcal{T} g_0) \omega_1 \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3, \\ F_5 &= 4R^4 l (\omega_{AdS_5} + \tilde{G} \omega_{S^5}), \end{aligned} \quad (2.11)$$

where

$$\begin{aligned} \tilde{G}^{-1} &= 1 + \tilde{\mathcal{P}} g_0, \\ \tilde{H} &= \csc^2 \xi + \hat{\sigma}^2 l^2 g_0, \\ \tilde{\mathcal{P}} &= \hat{\gamma}^2 \csc^2 \xi - 2\hat{\gamma} \hat{\sigma} l \cot \xi + \hat{\sigma}^2 l^2, \\ \tilde{\mathcal{Q}} &= \hat{\gamma} \csc^2 \xi - \hat{\sigma} l \cot \xi, \\ \tilde{\mathcal{T}} &= \hat{\gamma} l \cot \xi - \hat{\sigma} l^2. \end{aligned} \quad (2.12)$$

We study the Penrose limits of (2.4) along the $(J, 0, 0)$ geodesic and along the (J, J, J) geodesic.

The parametrization to take the Penrose limit along the $(J, 0, 0)$ geodesic is

$$\begin{aligned}\Xi_1 &:= f, \quad \rho = \frac{y}{\Xi_1^{1/4} R}, \quad \alpha = \frac{r}{\Xi_1^{1/4} R}, \\ t &= x^+ + \frac{x^-}{2\Xi_1^{1/2} R^2}, \quad \phi_1 = x^+ - \frac{x^-}{2\Xi_1^{1/2} R^2}, \\ r^2 &= \sum_{i=1}^4 (x^i)^2, \quad y^2 = \sum_{a=5}^8 (x^a)^2.\end{aligned}\tag{2.13}$$

By taking $R \rightarrow \infty$, we obtain the pp-wave geometry

$$\begin{aligned}ds^2 &= -2dx^+ dx^- - [y^2 + (1 + \mathcal{P})r^2](dx^+)^2 + dr^2 + r^2 d\tilde{\Omega}_3^2 + dy^2 + y^2 d\Omega_3^2, \\ e^\Phi &= \Xi_1 \tau_2^{-1}, \\ B_2 &= \frac{r^2}{\Xi_1^{1/2}} \mathcal{Q}(\cos^2 \theta dx^+ \wedge d\phi_2 - \sin^2 \theta dx^+ \wedge d\phi_3), \\ C_2 &= \frac{r^2}{\Xi_1^{1/2}} \mathcal{T}(\cos^2 \theta dx^+ \wedge d\phi_2 - \sin^2 \theta dx^+ \wedge d\phi_3), \\ C_4 &= -\frac{\tau_2}{\Xi_1} (y^4 dx^+ \wedge d\Omega_3 + r^4 dx^+ \wedge d\tilde{\Omega}_3),\end{aligned}\tag{2.14}$$

where

$$\begin{aligned}d\tilde{\Omega}_3^2 &= d\theta^2 + \cos^2 \theta d\phi_2^2 + \sin^2 \theta d\phi_3^2, \\ d\tilde{\Omega}_3 &= \cos \theta \sin \theta d\theta \wedge d\phi_2 \wedge d\phi_3.\end{aligned}\tag{2.15}$$

The parametrization to take the Penrose limit along the (J, J, J) geodesic [16] is

$$\begin{aligned}\Xi_2 &:= f + \frac{1}{3} \hat{\sigma}^2 \tau_2^2, \quad \theta_0 = \frac{\pi}{4}, \quad \alpha_0 = \arccos\left(\frac{1}{\sqrt{3}}\right), \\ \alpha &= \alpha_0 - \frac{x^2}{\Xi_2^{1/4} R}, \quad \theta = \theta_0 + \sqrt{\frac{3}{2}} \frac{x^1}{\Xi_2^{1/4} R}, \quad \rho = \frac{y}{\Xi_2^{1/4} R}, \\ \varphi^1 &= \sqrt{\frac{3 + \mathcal{P}}{2}} \frac{1}{\Xi_2^{1/4} R} \left(x^3 - \frac{1}{\sqrt{3}} x^4 \right), \quad \varphi^2 = \sqrt{\frac{2(3 + \mathcal{P})}{3}} \frac{x^4}{\Xi_2^{1/4} R}, \\ t &= x^+ + \frac{1}{2\Xi_2^{1/2} R^2} x^-, \quad \psi = x^+ - \frac{1}{2\Xi_2^{1/2} R^2} x^-, \end{aligned}\tag{2.16}$$

where the spherical coordinates and the torus coordinates are related by

$$\phi_1 = \psi - \varphi_2, \quad \phi_2 = \psi + \varphi_1 + \varphi_2, \quad \phi_3 = \psi - \varphi_1.\tag{2.17}$$

By taking $R \rightarrow \infty$ and shifting the coordinate x^- as $x^- \rightarrow x^- - \frac{\sqrt{3}}{\sqrt{3+\mathcal{P}}}(x^1x^3 + x^2x^4)$, we obtain the pp-wave geometry in the homogeneous plane wave form [17]²

$$\begin{aligned}
ds^2 &= -2dx^+dx^- + \frac{2\sqrt{3}}{\sqrt{3+\mathcal{P}}}(x^3dx^1 + x^4dx^2 - x^1dx^3 - x^2dx^4)dx^+ + \sum_{I=1}^8(dx^I)^2 \\
&\quad - \left[\sum_{a=5}^8 (x^a)^2 + \frac{4\mathcal{P}}{3+\mathcal{P}}((x^1)^2 + (x^2)^2) \right] (dx^+)^2, \\
e^\Phi &= \sqrt{\frac{3}{3+\mathcal{P}}} \Xi_2 \tau_2^{-1}, \\
B_2 &= \frac{\mathcal{Q}}{\sqrt{3}} \Xi_2^{-1/2} dx^3 \wedge dx^4 + \frac{2\mathcal{Q}}{\sqrt{3+\mathcal{P}}} \Xi_2^{-1/2} dx^+ \wedge (x^2dx^3 - x^1dx^4) \\
&\quad - \frac{2\sqrt{3}}{3} \hat{\sigma} \tau_2 \Xi_2^{-1/2} dx^+ \wedge (x^2dx^1 - x^1dx^2), \\
C_2 &= \frac{\mathcal{T}}{\sqrt{3}} \Xi_2^{-1/2} dx^3 \wedge dx^4 + \frac{2\mathcal{T}}{\sqrt{3+\mathcal{P}}} \Xi_2^{-1/2} dx^+ \wedge (x^2dx^3 - x^1dx^4) \\
&\quad - \frac{2\sqrt{3}}{3} \hat{\gamma} \tau_2 \Xi_2^{-1/2} dx^+ \wedge (x^2dx^1 - x^1dx^2), \\
C_4 &= 4R^4 \tau_2 \omega_4 \\
&\quad + 2\tau_2 \Xi^{-1} \left(1 - \frac{1}{3} \mathcal{T} \hat{\sigma} \right) dx^+ \wedge (x^2dx^1 \wedge dx^3 \wedge dx^4 - x^1dx^2 \wedge dx^3 \wedge dx^4). \quad (2.18)
\end{aligned}$$

3 Giant graviton on the deformed pp-wave

We study giant gravitons on the deformed pp-wave (2.14). A static gauge for a brane which wraps the (θ, ϕ_2, ϕ_3) directions is

$$\sigma^0 = \tau, \quad \sigma^1 = \theta, \quad \sigma^2 = \phi_2, \quad \sigma^3 = \phi_3, \quad (3.1)$$

and

$$X^+ = \lambda\tau, \quad X^- = \mu\tau. \quad (3.2)$$

The fields on the three-sphere (2.15) can be parameterized as

$$\begin{aligned}
X^1 &= r \cos \theta \cos \phi_2, \quad X^2 = r \sin \theta \cos \phi_3, \\
X^3 &= r \cos \theta \sin \phi_2, \quad X^4 = r \sin \theta \sin \phi_3. \quad (3.3)
\end{aligned}$$

² ω_1 in (2.3) is solved as $\omega_1 = \frac{1}{R^2} \Xi_2^{-1/2} \left[\left(\frac{\sqrt{3}}{9} - \zeta \right) x^1 dx^2 - \zeta x^2 dx^1 \right] + \mathcal{O}(R^{-3})$. $\zeta = \frac{\sqrt{3}}{18}$ is chosen, which is consistent with [13].

We turn off the fields on AdS_5

$$X^a = 0, \quad (a = 5, 6, 7, 8). \quad (3.4)$$

A D3-brane is described by the Dirac-Born-Infeld action and the Wess-Zumino term³

$$\begin{aligned} S &= S_{\text{DBI}} + S_{\text{WZ}} \\ &= -T_3 \int d^4\sigma e^{-\Phi} \sqrt{-\det P[g - B_2]} - T_3 \int \sum P[C_q \wedge e^{-B_2}]. \end{aligned} \quad (3.5)$$

P denotes the pullback of the spacetime field to the brane worldvolume. The D3-brane action in the deformed geometry (2.14) is

$$\begin{aligned} S &= -T_3 \int d^4\sigma \frac{\tau_2}{\Xi_1} \sqrt{-\det P[g - B_2]} - T_3 \int P[C_4] \\ &= -\frac{2\pi^2 T_3 \tau_2}{\Xi_1} \int d\tau \left[r^3 \sqrt{2\lambda\mu + \lambda^2 r^2 (1 + \hat{\sigma}^2 \tau_2^2 \Xi_1^{-1})} - \lambda r^4 \right]. \end{aligned} \quad (3.6)$$

The action does not depend on γ while it depends on σ as well as τ_1 and τ_2 .

The lightcone momentum⁴ of the D3-brane is

$$P^+ = -\frac{\delta L}{\delta \mu} = \frac{M\lambda r^3}{\Xi_1 \sqrt{2\lambda\mu + \lambda^2 r^2 (1 + \hat{\sigma}^2 \tau_2^2 \Xi_1^{-1})}}, \quad (3.7)$$

and the lightcone Hamiltonian is

$$P^- = H_{lc} = -\frac{\delta L}{\delta \lambda} = \frac{Mr^3}{\Xi_1} \left[\frac{\mu + \lambda r^2 (1 + \hat{\sigma}^2 \tau_2^2 \Xi_1^{-1})}{\sqrt{2\lambda\mu + \lambda^2 r^2 (1 + \hat{\sigma}^2 \tau_2^2 \Xi_1^{-1})}} - r \right], \quad (3.8)$$

where $M := 2\pi^2 \tau_2 T_3$. The lightcone Hamiltonian can be written as

$$H_{lc} \sim \frac{M^2 r^6}{2\Xi_1^2 P^+} + \frac{P^+ (1 + \hat{\sigma}^2 \tau_2^2 \Xi_1^{-1}) r^2}{2} - \frac{Mr^4}{\Xi_1}. \quad (3.9)$$

For $0 \leq \hat{\sigma} < \frac{\sqrt{\Xi_1}}{\sqrt{3}\tau_2}$, the Hamiltonian is extremized at

$$r_0 = 0, \quad r_{\pm} = \sqrt{\frac{P^+ \Xi_1}{3M} \left(2 \pm \sqrt{1 - 3\hat{\sigma}^2 \tau_2^2 \Xi_1^{-1}} \right)}. \quad (3.10)$$

³We choose the minus sign for the Wess-Zumino term as it is done in [12] since it is consistent with the conventions of [2, 3].

⁴The conjugate momenta are defined by $P_{\pm} = \frac{\delta L}{\delta(\partial_{\tau} X^{\pm})}$. The upper index and the lower index are related by $P^{\pm} = -P_{\mp}$.

The lightcone Hamiltonian has local minima at $r = r_0$ and $r = r_+$, and a local maximum at $r = r_-$. The radii do not depend on γ while they depend on σ as well as the axion-dilaton parameters τ_1 and τ_2 . The corresponding lightcone energies are

$$\begin{aligned} E_0 &= 0, \\ E_{\pm} &= \frac{(P^+)^2 \Xi_1}{27M} \left[1 + 9\hat{\sigma}^2 \tau_2^2 \Xi_1^{-1} \mp (1 - 3\hat{\sigma}^2 \tau_2^2 \Xi_1^{-1})^{3/2} \right]. \end{aligned} \quad (3.11)$$

For $\hat{\sigma} = 0$, we have $E_+ = E_0$, i.e., the giant graviton is degenerate with the point graviton. For $0 < \hat{\sigma} < \frac{\sqrt{\Xi_1}}{\sqrt{3}\tau_2}$, we have $E_+ > E_0$, i.e., the degeneracy is lifted. The energy of the point graviton is less than the energy of the giant graviton. Therefore the giant graviton becomes energetically unfavorable. For $\hat{\sigma} = \frac{\sqrt{\Xi_1}}{\sqrt{3}\tau_2}$, we have $r_+ = r_-$ and $E_+ = E_-$, i.e., there is a saddle point at $r = r_{\pm}$. The lightcone Hamiltonian has one minimum at $r = r_0$. For $\hat{\sigma} > \frac{\sqrt{\Xi_1}}{\sqrt{3}\tau_2}$, the giant graviton disappears. The result is consistent with the results of [13, 14].

The lightcone Hamiltonian in the Penrose limit of (2.11) is obtained by substituting $\Xi_1 = \csc^2 \xi$ and $\tau = \tau_1 + i\tau_2 = il$. The lightcone Hamiltonian for $\xi = \frac{\pi}{3}$ in the units of $M = 1$ and $P^+ = 1$ is plotted in Figure 1. It is qualitatively the same as the one plotted in [14]. The lightcone Hamiltonian for $\hat{\sigma}l = \frac{1}{3}$ in the units of $M = 1$ and $P^+ = 1$ is plotted in Figure 2. As the intersection angle ξ decreases, the minimum value of the lightcone Hamiltonian increases.

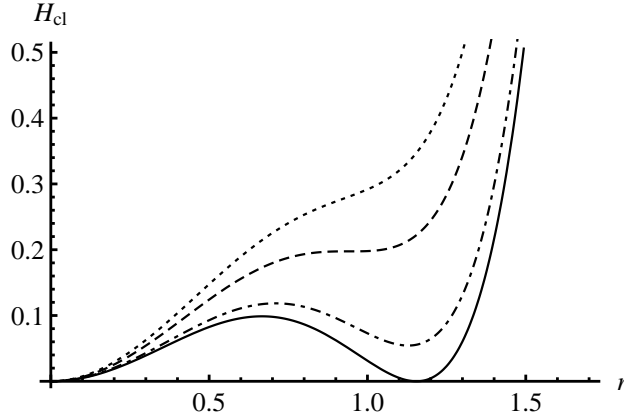


Figure 1: Lightcone Hamiltonian with $\xi = \frac{\pi}{3}$ as a function of r . $\hat{\sigma}l = 0$ (solid), $\hat{\sigma}l = \frac{1}{3}$ (dot-dashed), $\hat{\sigma}l = \frac{2}{3}$ (dashed), $\hat{\sigma}l = \frac{2.5}{3}$ (dotted), $\Xi_1 = \csc^2 \xi$, $\tau_2 = l$, $M = 1$ and $P^+ = 1$.

We examine the spectrum of small fluctuations about the giant graviton solution following the method of [13, 14, 18]. We fix the lightcone coordinates as

$$X^+ = \tau, \quad X^- = \nu\tau + \epsilon\delta x^-. \quad (3.12)$$

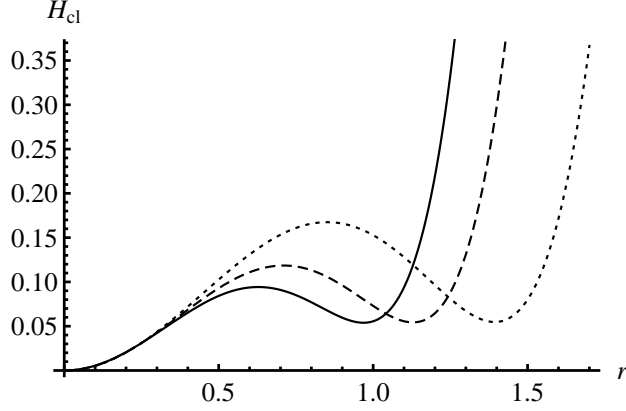


Figure 2: Lightcone Hamiltonian with $\hat{o}l = \frac{1}{3}$ as a function of r . $\xi = \frac{\pi}{2}$ (solid), $\xi = \frac{\pi}{3}$ (dashed), $\xi = \frac{\pi}{4}$ (dotted), $\Xi_1 = \csc^2 \xi$, $\tau_2 = l$, $M = 1$ and $P^+ = 1$.

The ansatz for the perturbed configuration is

$$\begin{aligned}
r &= r_0 + \epsilon \delta r, \\
X^1 &= r \cos \theta \cos \phi_2, \quad X^2 = r \sin \theta \cos \phi_3, \\
X^3 &= r \cos \theta \sin \phi_2, \quad X^4 = r \sin \theta \sin \phi_3, \\
X^a &= \epsilon \delta x^a, \quad (a = 5, \dots, 8).
\end{aligned} \tag{3.13}$$

The components of the pullback $D_{\mu\nu} = P[g - B_2]_{\mu\nu}$ up to the second order of ϵ are

$$\begin{aligned}
D_{\tau\tau} &= -2\nu - (1 + \mathcal{P})r_0^2 + \epsilon[-2\partial_\tau \delta x^- - 2(1 + \mathcal{P})r_0 \delta r] \\
&\quad + \epsilon^2[-\sum_a (\delta x^a)^2 - (1 + \mathcal{P})\delta r^2 + \sum_{I=i+a} (\partial_\tau \delta x^I)^2], \\
D_{\tau\theta} &= D_{\theta\tau} = -\epsilon \partial_\theta \delta x^- + \epsilon^2 \sum_{I=i+a} (\partial_\tau \delta x^I)(\partial_\theta \delta x^I), \\
D_{\tau\phi_2/\phi_2\tau} &= \mp \Xi_1^{-1/2} \mathcal{Q} r_0^2 \cos^2 \theta + \epsilon[-\partial_{\phi_2} \delta x^- \mp 2\Xi_1^{-1/2} \mathcal{Q} r_0 \cos^2 \theta \delta r] \\
&\quad + \epsilon^2[\sum_{I=i+a} \partial_\tau \delta x^I \partial_{\phi_2} \delta x^I \mp \Xi_1^{-1/2} \mathcal{Q} \cos^2 \theta \delta r^2], \\
D_{\tau\phi_3/\phi_3\tau} &= \pm \Xi_1^{-1/2} \mathcal{Q} r_0^2 \sin^2 \theta + \epsilon[-\partial_{\phi_3} \delta x^- \pm 2\Xi_1^{-1/2} \mathcal{Q} r_0 \sin^2 \theta \delta r] \\
&\quad + \epsilon^2[\sum_{I=i+a} \partial_\tau \delta x^I \partial_{\phi_3} \delta x^I \pm \Xi_1^{-1/2} \mathcal{Q} \sin^2 \theta \delta r^2],
\end{aligned} \tag{3.14}$$

$$D_{ij} = r_0^2 g_{ij} + 2\epsilon r_0 g_{ij} \delta r + \epsilon^2 \left[g_{ij} \delta r^2 + \sum_{I=1}^8 (\partial_i \delta x^I) (\partial_j \delta x^I) \right], \quad (i, j = \theta, \phi_2, \phi_3),$$

where the metric g_{ij} is defined as

$$g_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos^2 \theta & 0 \\ 0 & 0 & \sin^2 \theta \end{pmatrix}. \quad (3.15)$$

The D3-brane action is

$$\begin{aligned} \mathcal{S} &= \mathcal{S}_{\text{DBI}} + \mathcal{S}_{\text{WZ}} \\ &= -T_3 \int d^4 \sigma e^{-\Phi} \sqrt{-\det P[G - B]} - T_3 \int P[C_4 - C_2 \wedge B_2] \\ &= -T_3 \Xi_1^{-1} \tau_2 \int d\tau d^3 \sigma \sqrt{|g|} r_0^3 \left\{ \sqrt{2\nu + r_0^2 u} - r_0 \right\} \\ &\quad - \epsilon T_3 \Xi_1^{-1} \tau_2 \int d\tau d^3 \sigma \sqrt{|g|} \frac{r_0^3}{\sqrt{2\nu + r_0^2 u}} \left\{ \partial_\tau \delta x^- + \frac{2\delta r}{r_0} \left[3\nu + 2r_0^2 u - 2r_0 \sqrt{2\nu + r_0^2 u} \right] \right\} \\ &\quad - \epsilon^2 T_3 \Xi_1^{-1} \tau_2 \int d\tau d^3 \sigma \frac{r_0}{2\sqrt{2\nu + r_0^2 u}} \left\{ \sqrt{|g|} \left[30\nu + 28r_0^2 u - \frac{(6\nu + 4r_0^2 u)^2}{2\nu + r_0^2 u} \right. \right. \\ &\quad \left. \left. - 12r_0 \sqrt{2\nu + r_0^2 u} \right] \delta r^2 + \sqrt{|g|} r_0^2 \sum_a (\delta x^a)^2 - \sum_I \delta x^I \left[(2\nu + r_0^2 u) \partial_i (\sqrt{|g|} g^{ij} \partial_j) \right. \right. \\ &\quad \left. \left. + (\partial_{\phi_2} - \partial_{\phi_3}) (\sqrt{|g|} \Xi_1^{-1} \mathcal{Q}^2 r_0^2) (\partial_{\phi_2} - \partial_{\phi_3}) - r_0^2 \partial_\tau (\sqrt{|g|} \partial_\tau) \right] \delta x^I \right. \\ &\quad \left. + \sqrt{|g|} \frac{4(3\nu + r_0^2 u)}{2\nu + r_0^2 u} r_0 \delta r \delta_\tau \delta x^- - \delta x^- \partial_i (\sqrt{|g|} g^{ij} \partial_j) \delta x^- \right. \\ &\quad \left. + \frac{r_0^2}{2\nu + r_0^2 u} \delta x^- \partial_\tau (\sqrt{|g|} \partial_\tau) \delta x^- \right\}, \end{aligned} \quad (3.16)$$

where

$$u = 1 + \hat{\sigma}^2 \tau_2^2 \Xi_1^{-1}. \quad (3.17)$$

In the first order in ϵ , $\partial_\tau \delta x^- = 0$ as the endpoints in τ are fixed. From the second term, which is proportional to δr we get a constraint⁵

$$\nu_\pm = \frac{2r_0^2}{9} \left[-1 - 3\hat{\sigma}^2 \tau_2^2 \Xi_1^{-1} \pm \sqrt{1 - 3\hat{\sigma}^2 \tau_2^2 \Xi_1^{-1}} \right]. \quad (3.18)$$

⁵The constraint is also obtained from (3.7) and (3.10) with $\lambda := 1$ and $\mu := \nu$.

ν_+ minimizes the action.

To find the spectrum we decompose the solution as

$$\delta x^I = \delta \tilde{x}^I e^{-i\omega\tau} Y_{l,\alpha}. \quad (3.19)$$

$Y_{l,\alpha}$ are four-dimensional spherical harmonics which satisfy

$$\frac{1}{\sqrt{|g|}} \partial_i (\sqrt{|g|} g^{ij} \partial_j) Y_{l,\alpha} = -q_l Y_{l,\alpha}, \quad q_l = l(l+2). \quad (3.20)$$

Due to the term $\sim \mathcal{Q}^2 [(\partial_{\phi_2} - \partial_{\phi_3}) \delta x^I]^2$ in the fifth line of the action (3.16) the degeneracy of the spherical harmonics is lifted. The spherical harmonics are diagonalized as

$$\left(\frac{\partial}{\partial \phi_2} - \frac{\partial}{\partial \phi_3} \right)^2 Y_{l,\alpha} = -\alpha^2 Y_{l,\alpha}. \quad (3.21)$$

The spectrum in the X^a , ($a = 5, \dots, 8$), directions is

$$\omega_a^2 = 1 + \Xi_1^{-1} \mathcal{Q}^2 \alpha^2 + \frac{1}{9} \left(2 + \sqrt{1 - 3\Xi_1^{-1} \hat{\sigma}^2 \tau_2^2} \right)^2 q_l. \quad (3.22)$$

The radial direction and the null direction X^- are coupled. The equations of motion are

$$\begin{aligned} s &:= 1 - 3\Xi_1^{-1} \hat{\sigma}^2 \tau_2^2, \\ \left[\frac{8}{3} (\sqrt{s} - s) + \frac{1}{9} (2 + \sqrt{s})^2 q_l + \Xi_1^{-1} \mathcal{Q}^2 \alpha^2 - \omega^2 \right] \delta \tilde{r} - i \frac{\omega}{r_0} \left(\frac{6\sqrt{s}}{2 + \sqrt{s}} \right) \delta \tilde{x}^- &= 0, \\ i \frac{\omega}{r_0} \left(\frac{6\sqrt{s}}{2 + \sqrt{s}} \right) \delta \tilde{r} + \frac{1}{r_0^2} \left[q_l - \frac{9}{(2 + \sqrt{s})^2} \omega^2 \right] \delta \tilde{x}^- &= 0. \end{aligned} \quad (3.23)$$

The spectrum is

$$\begin{aligned} t &:= \frac{4\sqrt{s}(2 + \sqrt{s})}{3} + \Xi_1^{-1} \mathcal{Q}^2 \alpha^2, \\ \omega_{\pm}^2 &= \frac{t}{2} + \left(\frac{2 + \sqrt{s}}{3} \right)^2 q_l \pm 2 \sqrt{\frac{t^2}{16} + s \left(\frac{2 + \sqrt{s}}{3} \right)^2} q_l. \end{aligned} \quad (3.24)$$

ω_+^2 's are positive definite while ω_-^2 's are positive semidefinite. A zero mode occurs when $l = 0$. The spectrum is independent of the size r_0 . The spectrum depends on the marginal deformation parameters γ and σ as well as the axion-dilaton parameters τ_1 and τ_2 . There is no complex frequency. When $\hat{\sigma} \neq 0$, giant gravitons are not energetically favorable but the spectrum of small fluctuations shows that the giant gravitons are perturbatively stable. The result is consistent with the results of [13, 14].

4 Discussion

We have studied the Penrose limits of the complex marginal deformation of $AdS_5 \times S^5$ which incorporates the $SL(2, \mathbb{R})$ symmetry of type IIB theory and have presented the pp-wave geometries along the $(J, 0, 0)$ geodesic and along the (J, J, J) geodesic. We have shown that giant gravitons on the $(J, 0, 0)$ pp-wave depend the parameter σ as well as the axion-dilaton parameters. Giant gravitons exist up to a critical value of σ , which depends on the axion-dilaton parameters. The spectrum of small fluctuations about the giant graviton solution is obtained. The giant gravitons are energetically unfavorable but perturbatively stable.

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